**Self-assessment answers: 15 Complex numbers**

**1.** With *z* = *x* + i*y*, *z*\* = *x* − i*y*

⇒ *z* + 2*z*\* = 3*x* – i*y* = −1 + 4i

⇒ *x* = , *y* = −4

⇒ *z* =  4i*[5 marks]*

**2.** (cis *θ*)*n* = cis *nθ* and 

∴ 

**= ***[5 marks]*

**3.** Let *z* = *r*ei*θ*,

⇒ z3 = *r*3e3i*θ*

|−4 + 4| = 8

Arg(−4 +4) = arctan(−) = −

∴ *r*3e3i*θ* = 8e –i*π* ⁄ 3

⇒ *r* = 2, *θ* =  = 

⇒ *z* = 2e −7i*π* ⁄ 3, 2e –i*π* ⁄ 3, 2e 5i*π* ⁄ 3*[8 marks]*

**4.** (a) By De Moivre's theorem, (cos *θ* + i sin *θ*)*n* = cos *nθ* + i sin *nθ*.

⇒ *zn* – *z*−*n* = cos *nθ* + i sin *nθ* – (cos *nθ* – i sin *nθ*) = 2i sin *nθ*

(b) (*z* – *z*−1)5 = *z*5 – 5*z*3 + 10*z* – 10*z*−1 + 5*z*−3 – *z*−5

= (*z*5 – *z*−5) – 5(*z*3 – *z*−3) + 10(*z* – *z*−1)

(c) But by (a), (*z* – *z*−1)5 also equals (2i sin *θ*)5 = 32i sin5 *θ*.

∴ 32i sin5 *θ* = (*z*5 – *z*−5) – 5(*z*3 – *z*−3) + 10(*z* – *z*−1)

But by (a), (*zn* – z−*n*) = 2i sin *nθ*.

∴ 32i sin5 *θ* = 2i sin 5*θ* – 10i sin 3*θ* + 20i sin *θ*

⇒ sin5 *θ* = sin 5*θ* − sin 3*θ* + sin *θ**[12 marks]*